Derivatives of Logarithmic Functions:

$$\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}$$

When $a = e$ we have

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\log_a f(x)\right) = \frac{f'(x)}{f(x) \ln a}$$

$$\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}$$

Ex: Differentiate

1. $f(x) = \ln(\sin x)$

2. $y = w\log_3(w^2 + 1)$

3. $y = \ln\dfrac{1 - x^2}{\cos x}$

4. $y = \ln(\ln x)$

Ex: if $f(x) = \ln|x|$, find $f'(x)$

Ex: if $y = \dfrac{(x^2 + 1)^3 \sin^3 x}{e^{\cot x} \tan x}$ **find** $\dfrac{dy}{dx}$

Logarithmic differentiation Method:

1.   Take natural logarithms for both sides of the equation $y = f(x)$ and use the Laws of logarithms to simplify

2.   Differentiate implicitly with respect to x

3.   Solve the resulting equation for y'

The Pour Rule: If n is any real number and $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Ex: Differentiate

1. $y = 3^{\sqrt{2}}$

2. $y = \left[x^6 + 3x + \sin x \right]^{100}$

3. $y = 2^{\sin x + \cos(e^x)}$

4. $y = x^x$

5. $y = (x+1)^{\sin x}$

6. $g(s) = (s^2 + 1)e^{(s + \sin s)}$

7. $y = (\ln u)^{\sin u} (\tan u + 3)^u$

8. $y = (x+1)^2 \sin(\cos x)\sqrt{\tan x}$

9. $y = (x+1)^2 + \sin(\cos x) + \sqrt{\tan x}$

$$\lim_{x \to 0}(1+x)^{\frac{1}{x}} = e$$