Strategy and Tactics for Integration

Table of Integration Formulas.

1. $\int x^n\ dx$	$=\frac{x^{n+1}}{n+1}+C\ (n\neq -1)$	2. $\int \frac{1}{x}\ dx$	$=\ln\|x\|+C$
3. $\int e^x\ dx$	$=e^x+C$	4. $\int a^x\ dx$	$=\frac{a^x}{\ln a}+C\ (a>0)$
5. $\int \sin x\ dx$	$=-\cos x+C$	6. $\int \cos x\ dx$	$=\sin x+C$
7. $\int \sec^2 x\ dx$	$=\tan x+C$	8. $\int \csc^2 x\ dx$	$=-\cot x+C$
9. $\int \sec x\tan x\ dx$	$=\sec x+C$	10. $\int \csc x\cot x\ dx$	$=-\csc x+C$
11. $\int \sec x\ dx$	$=\ln\|\tan x+\sec x\|+C$	12. $\int \csc x\ dx$	$=\ln\|\csc x-\cot x\|+C$
13. $\int \tan x\ dx$	$=\ln\|\sec x\|+C$	14. $\int \cot x\ dx$	$=\ln\|\sin x\|+C$
15. $\int \sinh x\ dx$	$=\cosh x+C$	16. $\int \cosh x\ dx$	$=\sinh x+C$
17. $\int \frac{dx}{x^2+a^2}$	$=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$	18. $\int \frac{dx}{\sqrt{a^2-x^2}}$	$=\sin^{-1}\left(\frac{x}{a}\right)+C$
19. $\int \frac{dx}{\sqrt{x^2+a^2}}$	$=\sinh^{-1}\left(\frac{x}{a}\right)+C$	20. $\int \frac{dx}{\sqrt{x^2-a^2}}$	$=\cosh^{-1}\left(\frac{x}{a}\right)+C$

If you are given an integral not from the previous table try the following seven-step strategy

(1) **Simplification :** If it possible to simplify the integration using algebraic manipulation or trigonometric identities. Examples:

(i)

$$\int \sin 3x\cos 2x\ dx=\frac{1}{2}\int(\sin x+\sin 5x)\ dx.$$

(ii)

$$\int \frac{\tan x}{\sec^2 x}\ dx=\int \frac{\sin x}{\cos x}\cos^2 x\ dx=\int \sin x\cos x\ dx=\frac{1}{2}\int \sin 2x\ dx.$$

(2) **Using the table for a function instead of x [Substitution] :**

If the integrand is a function whose differential also occurs, apart from the constant factor. Examples:

(i)

$$\int \frac{\cos x}{1+3\sin x}\,dx \overset{\text{apply formula 2 in the table}}{=} \frac{1}{3}\int \frac{3\cos x}{1+3\sin x}\,dx = \ln|1+3\sin x| + C.$$

(ii)

$$\int \frac{x}{4+x^4}\,dx \overset{\text{apply formula 17 in the table}}{=} \frac{1}{2}\int \frac{2x}{2^2+(x^2)^2}\,dx = \frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right) + C.$$

(3) **Integration by part:**

Use this when the integrand is the product of polynomial(could be constant) and trigonometric(exponential,logarithmic,trigonometric inverse) function, or

the integrand is the product of trigonometric function and exponential function. see section 8.1

Examples

(i)

$$\int x^4\cos x\ dx,\ \int \ln x\ dx,\ \int x^5e^x dx,\ \int x^2\tan^{-1}x\ dx.$$

(ii)

$$\int e^x\cos x\ dx,\ \int e^x\sin x\ dx.$$

(4) **Trigonometric Functions :**

Use this method when the integrand is the product of powers of $\sin x$ and $\cos x$,of $\tan x$ and $\sec x$, or of $\csc x$ and $\cot x$. see section 8.2 Examples:

$$\int \sin^3 x\cos^2 x\ dx,\ \int \tan^3 x\sec^5 x\ dx,\ \int \cot^2 x\csc^4 x\ dx.$$

(5) **Trigonometric Substitution :**

Use this method in case of an integral involving a^2+x^2, a^2-x^2, or x^2-a^2. see section 8.3 Examples:

$$\int \frac{x^2}{\sqrt{x^2+4}}\,dx,\ \int \frac{x^2}{\sqrt{9-x^2}}\,dx,\ \int \frac{e^x}{\sqrt{e^{2x}-4}}\,dx.$$

(6) **Completing the Square :**

Use this method in case of an integral involving $\sqrt{ax^2+bx+c}$, or if the integral of the form $\int \frac{1}{(ax^2+bx+c)^m}\,dx$ and ax^2+bx+c is irreducible . If the integral is not easy you may need to try trigonometric substitution after completing the square. see section 8.3 Examples :

$$\int \frac{1}{\sqrt{x^2+2x+10}}\,dx,\ \int \sqrt{5+4x-x^2}\,dx,\ \int \frac{e^x}{\sqrt{e^{2x}-6e^x-5}}\,dx,\ \int \frac{1}{x^2+x+1}\,dx.$$

(7) **Partial Fraction and Long Division :**

Use this method in case of an integral is a quotient of polynomials.

(i) If the degree of the numerator is bigger than or equal to the degree of the denominator use the long division first and partial fraction later. Examples:

$$\int \frac{x^2}{\sqrt{x+10}}\,dx,\ \int \frac{x^4}{x^2+1}\,dx,\ \int \frac{x}{3x+1}\,dx.$$

(ii) If the degree of the numerator is less than the degree of the denominator use the partial fraction.

Examples:

$$\int \frac{1}{x^2-5x+6}\,dx,\ \int \frac{2x+1}{x^4-1}\,dx,\ \int \frac{x}{(x+1)^2(x^2+1)^3}\,dx.$$

Reduction Formulas :

(1)

$$\int \cos^n x\ dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\ dx.$$

(2)

$$\int \sin^n x\ dx = \frac{-1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\ dx.$$

(3)

$$\int \tan^n x\ dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\ dx, \quad n \geq 2.$$

(4)

$$\int \cot^n x\ dx = \frac{-1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\ dx, \quad n \geq 2.$$

(5)

$$\int \sec^n x\ dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\ dx, \quad n \geq 2.$$

(6)

$$\int \csc^n x\ dx = \frac{-1}{n-1}\csc^{n-2} x \cot x + \frac{n-2}{n-1}\int \csc^{n-2} x\ dx, \quad n \geq 2.$$

(7)

$$\int (\ln x)^n\ dx = x(\ln x)^n - n\int (\ln x)^{n-1}\ dx.$$